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Online Appendix to "Effects of a Money-financed Fiscal Stimulus Without Irredeemability of Money"*.

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A Non-policy Blocks

Households maximize its utility given by:

$$\sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(C_t, L_t, N_t; Z_t\right), \tag{A.1}$$

with $U\left(C_t, L_t, N_t; Z_t\right) \equiv \left(U\left(C_t, L_t\right) - V\left(N_t\right)\right) Z_t$ subject to a sequence of budget constraints:

$$P_t C_t + B_t + M_t = B_{t-1}\left(1 + i_{t-1}\right) + M_{t-1} + W_t N_t + D_t - P_t TR_t, \tag{A.2}$$

where C_t denotes consumption and N_t is employment.

The optimality conditions are given by:

$$U_{c,t} = \beta\left(1 + i_t\right)\Pi_{t+1} U_{c,t+1}, \tag{A.3}$$

$$\frac{W_t}{P_t} = \frac{V_{n,t}}{U_{c,t}}, \tag{A.4}$$

$$\frac{U_{l,t}}{U_{c,t}} = \frac{i_t}{1 + i_t}. \tag{A.5}$$

Profit maximization under perfect competition leads to a demand schedule as follows:

$$Y_t\left(j\right) = \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\epsilon}, \tag{A.6}$$

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where $Y_t(j)$ denotes the quantity of good $j \in [0,1]$. Note that the aggregator is given by:

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{A.7}$$

Each firm produces a differentiated good with a technology as follows:

$$Y_t(j) = N_t(j)^{1-\alpha}. \tag{A.8}$$

Each firm can reset the price of its good with probability $1-\theta$ in any given period, subject to the isoelastic demand schedule Eq.(A.6). The FONC for firms is given by:

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}} \right) Y_{t+k|t} \left(\tilde{P}_t - \mathcal{M} MC^n_{t+k|t} \right) \right] = 0, \tag{A.9}$$

with $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1}$ and:

$$Y_{t+k|t} \equiv \left(\frac{\tilde{P}_t}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}, \tag{A.10}$$

where $\tilde{P}_t$ denotes the price chosen by firms when they have a chance to change their prices, $MC^n_{t+k|t}$ denote the nominal marginal cost under nominal rigidities. Market clearing condition is given by:

$$Y_t(j) = C_t(j) + G_t(j), \tag{A.11}$$

Plugging Eq.(7) into Eq.(A.11) yields:

$$Y_t = C_t + G_t, \tag{A.12}$$

where we assume that aggregator of consumption and government expenditure is analogous to aggregator of output.

B Steady State and Equilibrium Dynamics

Steady state output and real balances, which are given by the system:

$$(1-\alpha) U_c = \mathcal{M} V_n N^{\alpha}, \tag{B.1}$$

$$\frac{U_l}{U_c} = \frac{\rho}{1+\rho}. \tag{B.2}$$

The equilibrium around the steady state can be approximated by the following system:

$$\hat{y}_t = \hat{c}_t + \hat{g}_t, \tag{B.3}$$

$$\hat{\xi}_t = \hat{\xi}_{t+1} + \hat{i}_t - \pi_{t+1} - \hat{\rho}_t, \tag{B.4}$$

$$\hat{\xi}_t = -\sigma \hat{c}_t + \upsilon \hat{l}_t, \tag{B.5}$$

$$\pi_t = \beta \pi_{t+1} - \kappa_t \hat{\mu}_t, \tag{B.6}$$

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\alpha+\varphi}{1-\alpha} \hat{y}_t, \tag{B.7}$$

$$\hat{l}_t = \hat{c}_t - \eta \hat{i}_t, \tag{B.8}$$

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t \Delta m_t, \tag{B.9}$$

as well as Eq.(10) with $\sigma \equiv -\frac{U_{cc}C}{U_c}$, $\varphi \equiv \frac{V_{nn}N}{V_n}$, $\upsilon \equiv \frac{U_{cl}L}{U_c}$, $\eta \equiv \frac{\epsilon_{lc}}{\rho}$, $\epsilon_{lc} \equiv -\frac{1}{h'}\frac{\rho}{1+\rho}V$ and $h\left(\frac{C}{L}\right) \equiv \frac{U_l}{U_c}$.

Eq.(B.3) results from log-linearizing Eq.(A.12). Eqs.(B.4) and (B.8) result from Eqs.(A.3) and (A.5), respectively. Eq.(B.5) is derived by log-linearizing $U_{c,t}$. Eq.(B.6) results from log-linearizing Eq.(A.9). Eq.(B.9) results from log-linearizing the marginal cost $MC_t = \frac{V_{n,t}}{U_{c,t}}\frac{N_t^{\alpha}}{1-\alpha}$. Eq.(B.9) results from log-linearizing the definition of the real money balance.

C Non-policy Blocks in a Two-country Economy

Infinitely lived households in country H maximize Eq.(A.1) subject to:

$$P_tC_t + B_{H,t} + \mathcal{E}_tB^*_{H,t} + M_t = B_{H,t-1}\left(1+i_{t-1}\right) + \mathcal{E}_tB^*_{H,t-1}\left(1+i^*_{t-1}\right) + M_{t-1} + W_tN_t - P_tTR_t,$$

where $B_{H,t}$ denotes the nominal country H's government debt held by households in country H and $B^*_{H,t}$ denotes the nominal country F's government debt held by households in country H. Note that $B_{H,t} + B_{F,t} = B_t$ where $B_{F,t}$ is the nominal country H's government debt held by households in country H.

Infinitely lived households in country F maximize the counterpart of Eq.(A.1) subject to the previous expression. Optimality conditions are given by Eqs.(A.3) to (A.5) and counterparts of them.

Financial markets are complete internationally so that $\mathcal{U}_{c,t} = \mathcal{U}^*_{c,t}$.

We assume the law of one price such that $P_t(h) = \mathcal{E}_tP^*_t(h)$ with $h \in [0,\nu)$ and its counterpart. Plugging those expressions into $P_{H,t} \equiv \left[\frac{1}{\nu}\int_0^{\nu} P_t(h)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ and its counterpart in country F, we have $P_{H,t} = \mathcal{E}_tP^*_{H,t}$ and $P_{F,t} = \mathcal{E}_tP^*_{F,t}$. Plugging previous expressions into the definition of the CPI, we have purchasing power parity condition (PPP) $P_t = \mathcal{E}_tP^*_t$.

Plugging the PPP into the definition of the real exchange rate $\mathcal{Q}_t \equiv \frac{\mathcal{E}_tP^*_t}{P_t}$, we have:

$$\mathcal{Q}_t = 1, \tag{C.1}$$

Combining Eq.(A.3), its counterpart in country F and Eq.(C.1), we have:

$$1 + i_t = \left(1+i^*_t\right)\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t},$$

which is the UIP.

Analogous to Eq.(A.6), demand schedule for generic good h is given by:

$$Y_t(h) = \left(\frac{P_t(h)}{P_{H,t}}\right)^{-\epsilon}. \tag{C.2}$$

Demand schedule for generic good f is given similarly. The aggregator is now given by:

$$Y_t \equiv \left[\left(\frac{1}{\nu}\right)^{\frac{1}{\epsilon}}\int_0^{\nu} Y_t(h)^{\frac{\epsilon-1}{\epsilon}}\,dh\right]^{\frac{\epsilon}{\epsilon-1}}, \tag{C.3}$$

instead of Eq.(A.7). The aggregator of Y^*_t is given similarly.

Domestic producers in country H has technology $Y_t(h) = N_t(h)^{1-\alpha}$ similar to Eq.(A.8). Its counterpart in country F is similarly.

Similar to Appendix A, each firm can reset the price of its good with probability $1-\theta$ in any given period. However, not $\tilde{P}_t$ but $\tilde{P}_{H,t}$ is chosen to reset so that the FONC for firms is given by:

$$\sum_{k=0}^{\infty} \theta^k \left[\Lambda_{t,t+k} \left(\frac{1}{P_{t+k}} \right) Y_{t+k|t} \left(\tilde{P}_{H,t} - \mathcal{M} MC^n_{t+k|t} \right) \right] = 0, \tag{C.4}$$

instead of Eq.(A.9). Note that Eq.(A.10) is replaced by $Y_{t+k|t} \equiv \frac{\tilde{P}_{H,t}}{P_{H,t+k}} Y_{t+k}$. There is a counterpart of Eq.(C.4) in country F.

Market clearing condition is given by Eq.(A.11). Plugging Eq.(C.2) into Eq.(A.11) yields:

$$Y_t = \mathcal{S}_t^{1-\nu} C_t^W + G_t, \tag{C.5}$$

where C_t^W is aggregate consumption in the whole economy. Counterpart of Eq.(C.5) in country F is analogous to Eq.(C.5).

D Steady State and Equilibrium Dynamics in a Two-country Economy

Steady state in a two-country economy is described by Eqs.(B.1) and (B.2), counterparts of Eqs.(B.1) and (B.2) and $\mathcal{Q} = 1$. Note that even in country F, steady state government expenditure is zero.

The equilibrium around the steady state can be approximated by Eq.(10), Eqs.(B.4) and (B.5), Eqs.(B.8) to (B.9) and counterparts of them. Instead of Eqs.(B.3), (B.6) and (B.7), following log-linearized equalities describe equilibrium dynamics:

$$\nu \hat{y}_t + (1-\nu)\hat{y}_t^* = \nu \hat{c}_t + (1-\nu)\hat{c}_t^* + \nu \hat{g}_t + (1-\nu)\hat{g}_t^*, \tag{D.1}$$

$$\pi_{H,t} = \beta \pi_{H,t+1} - \kappa \hat{\mu}_t, \tag{D.2}$$

$$\hat{\mu}_t = \hat{\xi}_t - \frac{\varphi + \alpha}{1-\alpha}\hat{y}_t - (1-\nu) s_t, \tag{D.3}$$

and counterparts of Eqs.(D.2) and (D.3). Eq.(D.1) results from log-linearizing Eq.(C.5) and its counterpart in country F. Eq.(C.4) results from log-linearizing Eq.(C.4). Eq.(D.3) results from log-linearizing the marginal cost in country H $MC_t = \frac{V_{n,t}}{U_{c,t}} \frac{P_t}{P_{H,t}} \frac{N_t^\alpha}{1-\alpha}$. In addition:

$$s_t = \hat{y}_t - \hat{y}_t^* - \hat{g}_t + \hat{g}_t^*, \tag{D.4}$$

$$\hat{\xi}_t = \hat{\xi}_t^* - \hat{\rho}_t + \hat{\rho}_t^*, \tag{D.5}$$

are essential to describe equilibrium dynamics. Eq.(D.4) results from log-linearizing Eq.(C.5) and its counterpart in country F. Eq.(D.5) results from log-linearizing $\mathcal{U}_{c,t} = \mathcal{U}^*_{c,t}$.